

# Critical Thinking

## Session One

*Illusions in Reasoning*

# The Missing Dollar Puzzle

3 people go into a motel. The clerk behind the desk tells them **"The room is \$30"** They each pay **\$10**. A little later, the clerk realises **the room is only \$25**, so he sends someone to the room with **\$5 in change**. On the way the underling **can't work out how to split \$5 evenly between 3 people**, so ends up giving each of them **\$1 and pockets the remainder**. This means that the guests each paid \$9 for the room, which is a total of \$27. If we add in the \$2 the attendant kept, it adds up to \$29.

So, where is the missing dollar?

# Two Problems with the 'Missing Dollar' Puzzle

## The First Problem:

The way the sum is presented in the story is wrong:

\$9 paid by each guest, so  $9 \times 3 = 27$

\$2 kept by the underling, so  $\$27 + \$2 = \$29$ .

*This leaves \$1 missing.*

Contrast the presentation of the story (a story about adding up) with 'Where did each dollar end up?':

\$30-5 ends up in the till, so \$25 ends up in the till.

\$1 is returned to each of the three guests, so that accounts for \$3.

\$2 ends up with the attendant.

So, there really is no missing dollar.

**Note:** 'the guests each paid \$9 for the room' is not a fact about where the money ended up, so it shouldn't be in the story.



# Why do we get fooled?

The 'Missing Dollar' puzzle is some sort of trick. How it manages to fool us is a bit of a mystery (and is best explained by psychologists).

# Sometimes people are ignorant of particular facts

## Example

People instinctively run upstairs in a fire, not realising perhaps that this puts them in more danger because the fire will spread upwards.

**Sometimes, people simply fail to use the information they have in a rational way.**

## Example

Even after hearing all the facts about interest rates and term deposits, people are often reluctant to change banks.

# School Report for Bruce Fairchild, Year 13, Junior Ganymede High School

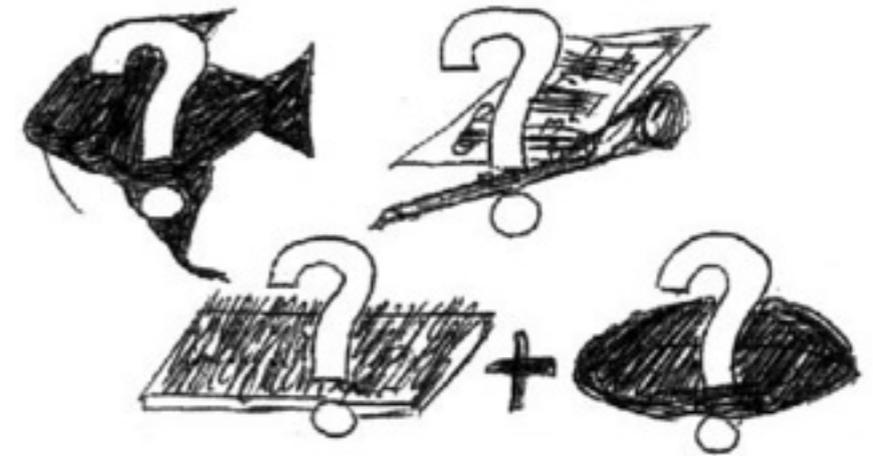
<b>Religious Education:</b>	Bruce is a diligent, if unremarkable, student whose understanding of Canon Law could well prove beneficial should he ever wish to sue the Church. Possibly a latent skeptic or a firm fundamentalist; not really sure (nor is he).
<b>Biology:</b>	Bruce's delight in dissecting rats is only made better by his thorough appreciation of anatomy and and the rightful place of the fish finger in class. Could possibly try harder to study the other aspects of the course.
<b>Chemistry:</b>	Likes to mix things up. Doesn't like to do his homework. Could benefit from reading the textbook and not sitting next to Sarah Cubbins.
<b>Physics:</b>	An unremarkable student. Shows no understanding of the quantum theories of gravity nor comprehends basic time asymmetry in regard to our temporal experiments. Likely to never go back and kill his own grandfather. A pity.
<b>Mathematics:</b>	Confuses subtraction with division three point five times out of Pi to the power of thirteen. Does like Lewis Carroll, which is something.
<b>English:</b>	Uses the language like a shovel. 'Nuff said.
<b>Astrology:</b>	With his grades in ascendance over Orion and the coursework in the House of Mars I can safely say that Bruce will meet an interesting person within the next three weeks. His lucky numbers are 13 and 67 and his special colour is blue.
<b>Woodwork:</b>	Very adept at fitting lintels and carving wood. Has great promise as a handyman.
<b>Music:</b>	I was fairly surprised to see how well Bruce took up reading sheet music seeing that he is a little tone deaf. Seems to have got over that problem though with careful finger work and a reliance on others to judge his performance. No remedial work required.

a. Bruce is a marine biologist

b. Bruce is a carpet layer

c. Bruce plays the oboe

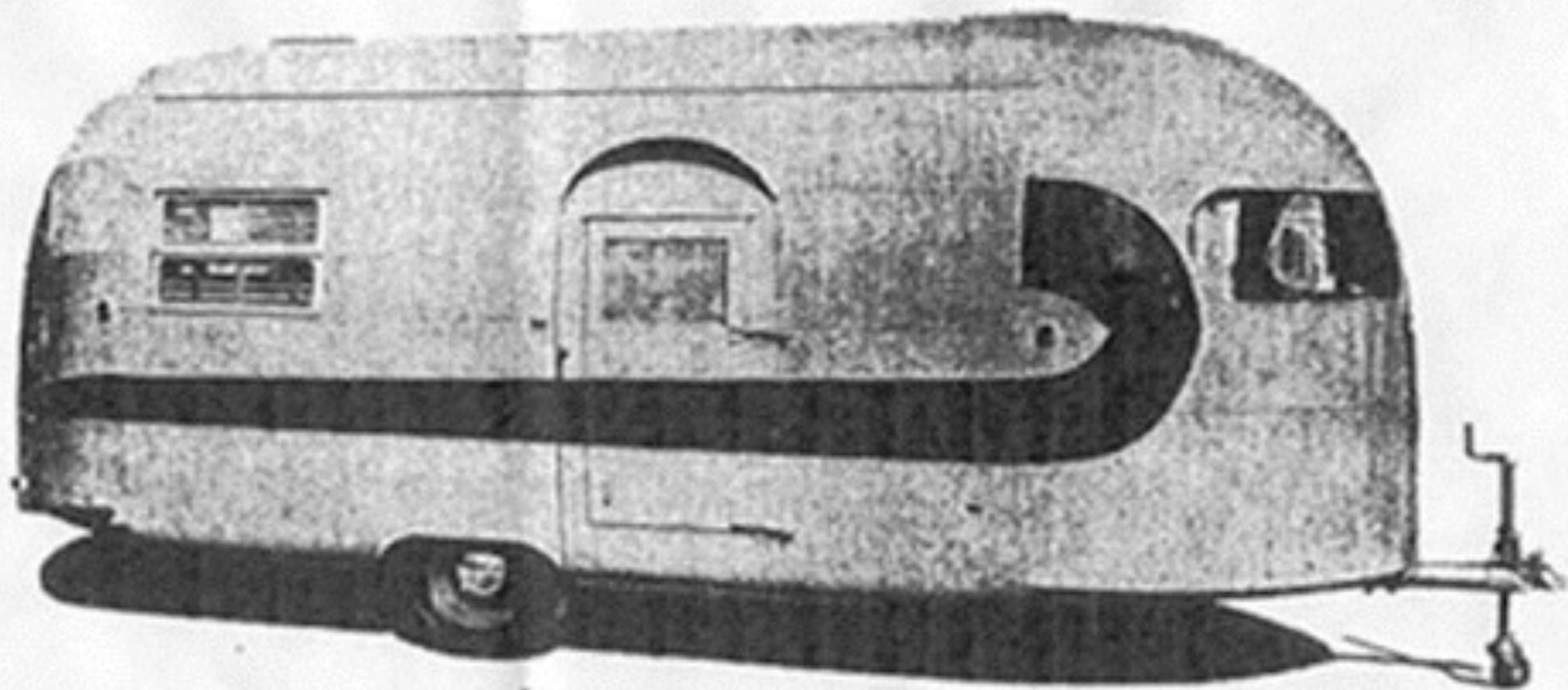
d. Bruce is a rugby coach and a carpet layer



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Pretend we know nothing about American geography.  
Could all these propositions be true at the same time?

- Boston is south of San Francisco
- Chicago is west of Boston
- New York is south of Boston
- Chicago is east of New York
- New York is north of San Francisco
- Washington D.C. is south of Chicago
- Washington D.C. is south of Washington D.C.

The last one is false. We can know this without knowing anything about the specifics of American geography

Can all but the last one be true?

Boston is south of San Francisco

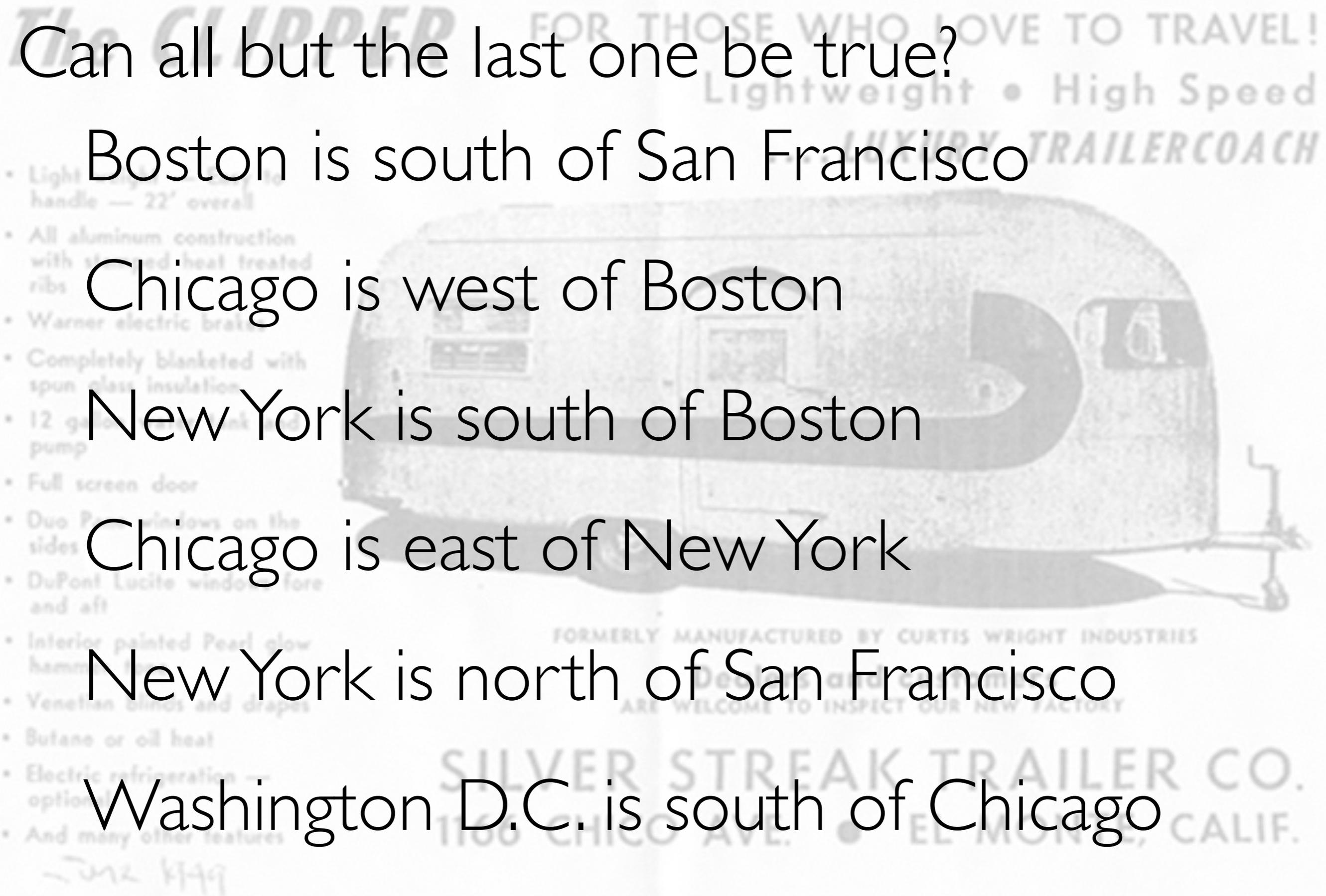
Chicago is west of Boston

New York is south of Boston

Chicago is east of New York

New York is north of San Francisco

Washington D.C. is south of Chicago



No. The first, third and fifth are in conflict. At least one of them must be false.

If Boston is south of San Francisco and New York is south of Boston, then New York cannot be north of San Francisco

## Moral

There are some things which we can know just by thinking about them.

# Arguments

Do the follow arguments entail or suggest the conclusion

## Case One

There are still a few cakes available.

Therefore,

There is some food.

*The first proposition entails the second.*

## Case Two

I've been up since about five.

Therefore,

I was up at five.

*The first proposition does not entail the second.*

## Case Three

I've had an espresso coffee and a hot chocolate today.

Therefore,

I've had some coffee today.

*The first proposition entails the second.*

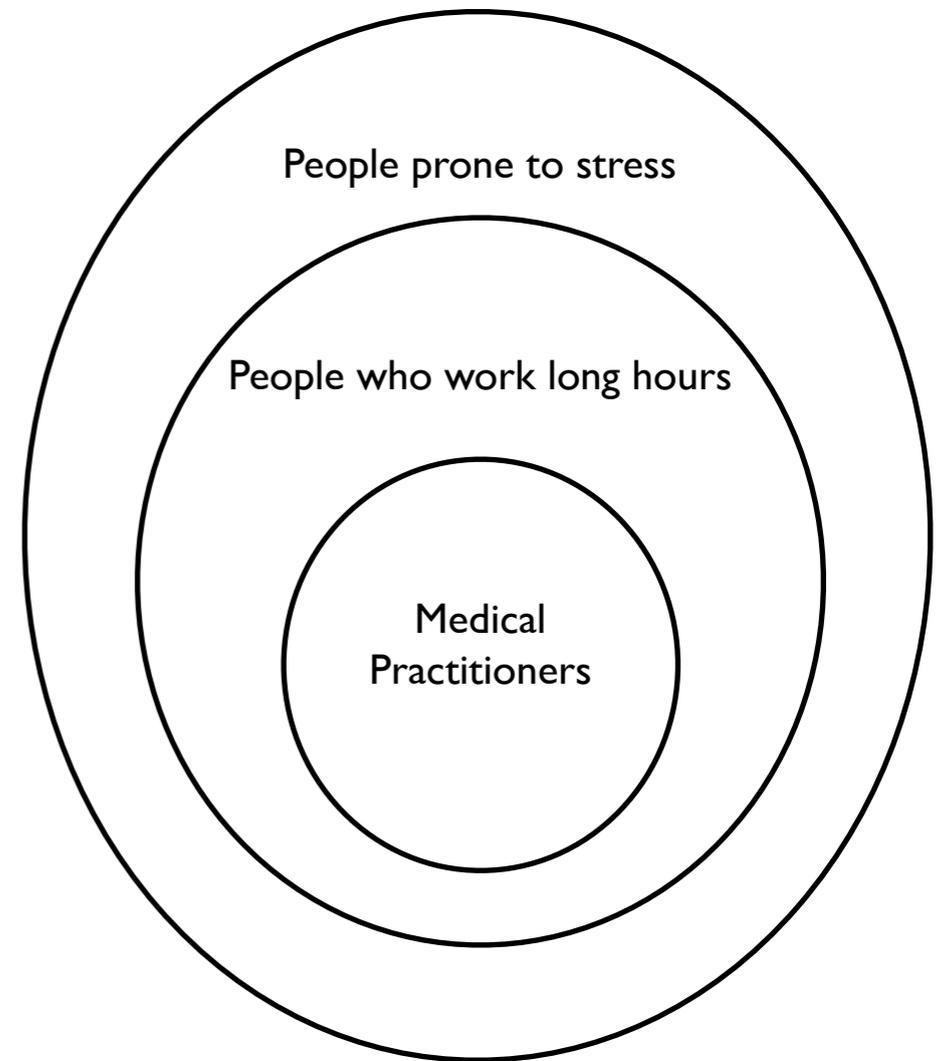
# Case Four

Every medical practitioner works long hours.

Anybody who works long hours is prone to stress.

Therefore,

Every medical practitioner is prone to stress.



*The premises entail the conclusion.*

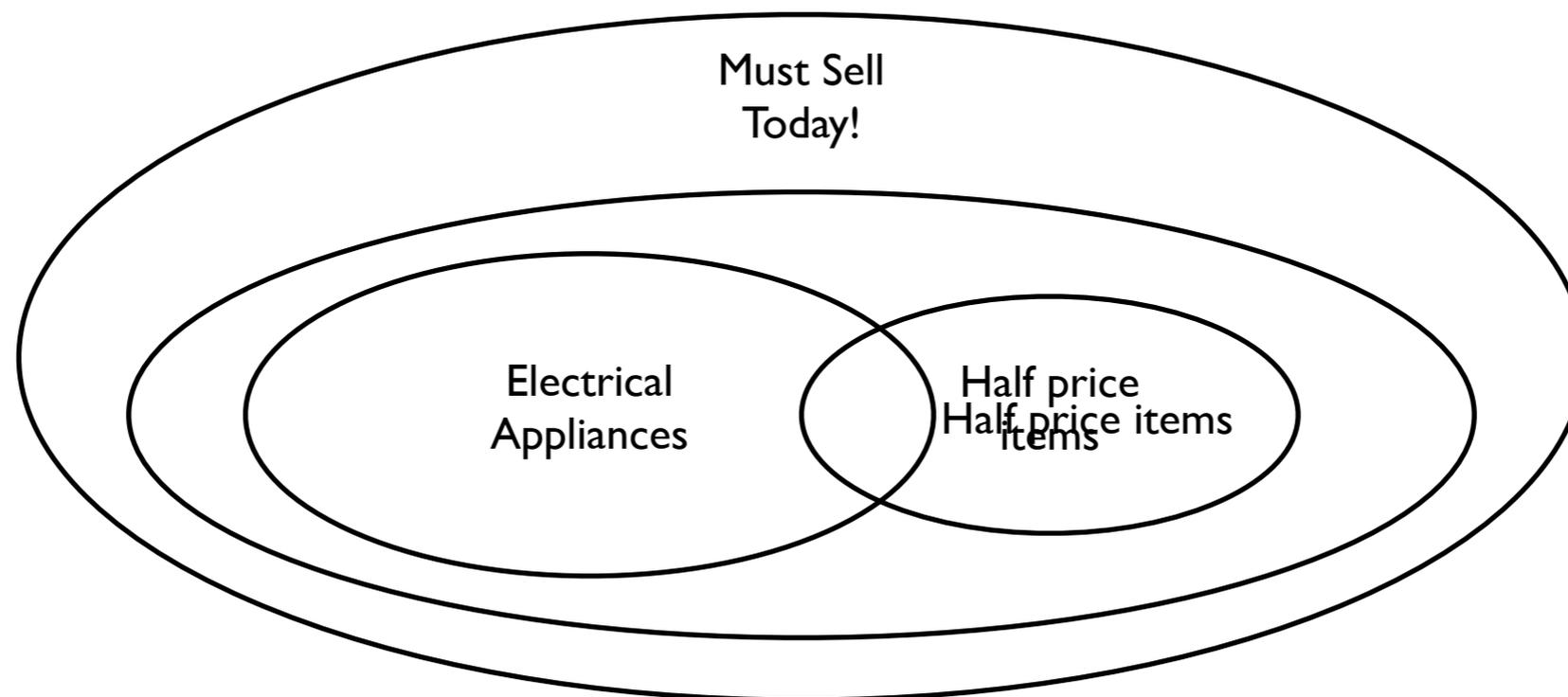
# Case Five

All electrical appliances must sell today.

Only some of the stuff that must sell today is half price.

Therefore,

Some, but not all, of the electrical appliances are half price.



# Example

P1. We have arrived home and discovered a dead blackbird in the middle of the floor.

P2. We have a cat.

P3. We have a cat-door (which the cat uses frequently)

P4. All the windows were closed until we came home

P5. Birds never got in here before we got the cat.

Together, the statements in this collection strongly suggest that:

C. Our cat brought the blackbird in.

# Entailment vs. Suggestion

1. Suppose that the premises are true.
2. Work out how probable the conclusion is following the premises.

Relationship	Probability of Conclusion Given the Premises
Entailment	1
Suggestion (weak - strong)	0.5 - 0.9999
Undermining	0.1 - 0.49999
Inconsistent	0

# Example 1

P1. I enjoy a stroll through Albert Park.

Therefore,

C. I never enjoy strolling through Albert Park.

Probability of Conclusion Given the Premises	Relationship
1	Entailment
0.5 - 0.9999	Suggestion
0.1 - 0.49999	Undermining
0	Inconsistent

# Example 2

P1. Eating lots of fruit and vegetables makes you healthy.

P2. You must do anything that makes you healthy.

Therefore,

C. You must eat lots of fruit and vegetables.

Probability of Conclusion Given the Premises	Relationship
1	<b>Entailment</b>
0.5 - 0.9999	Suggestion
0.1 - 0.49999	Undermining
0	Inconsistent

# Example 3

P1. I've just won the major award that I've had my heart set on for twenty years!

Therefore,

C. I'm in an especially bad mood!

Probability of Conclusion Given the Premises	Relationship
1	Entailment
0.5 - 0.9999	Suggestion
0.1 - 0.49999	<b>Undermining</b>
0	Inconsistent

# Example 4

P1. My father is a plumber.

Therefore,

C. My father has a van.

Probability of Conclusion Given the Premises	Relationship
1	Entailment
0.5 - 0.9999	<b>Suggestion</b>
0.1 - 0.49999	Undermining
0	Inconsistent

# Beware!

Don't treat strong suggestion as entailment!

## Example

Suppose you hear:

The rioters were throwing bottles into the street all day.  
The police and para-military were kept busy sweeping  
milk and broken glass off the roadway.

Question: What was in the bottles thrown by the rioters?

# Are these two claims inconsistent?

## Example One

1. My WiFi was down this morning.
2. My WiFi was up-and-running for the whole of this morning.

*These two propositions are inconsistent.*

# Example Two

1. The Prime Minister said that he had complete confidence in Judith Collins.

2. The Prime Minister said that he did not have complete confidence in Judith Collins.

*These propositions are not inconsistent. He could have said both these things.*

# Example Three

1. The Finance Minister said that there was not enough wiggle room in the forecasted budget surplus to allow for tax cuts in the short term.

2. The Finance Minister did not say that there was not enough wiggle room in the forecasted budget surplus to allow for tax cuts in the short term.

*These statements are inconsistent. Either he said it or he didn't say it. Both can't be true.*

# Evaluating Arguments

To find out whether an argument is good, we must perform two tasks.

**1. The Logical Task:** We must suppose that the premises are all true and then determine how probable the conclusion is given that supposition.

**2. The Material Task:** We must learn whether the premises are true or, at any rate, we must decide how plausible they are.

If an argument fails the logical task OR the material task OR both, it is not a good argument and we should reject it.

*The conclusion, however, could still be true.*

# Performing the Material Task

Do we have good reasons for believing the premises?

If one of the premises says:

22% of all infants are born with a 56% chance of developing breast cancer

then we must do research into statistics and epidemiology.

The main issue is whether the premises are plausible.

# Examples of *Plausible* Premises

The University of Auckland has many students doing a BA degree.

John Key is currently the Prime Minister of New Zealand.

Temperatures in Saudi Arabia tend to be hotter on average than temperatures in Canada.

If a premise is controversial, then it is not plausible. We should not accept an argument if it has a controversial premise.

## **Examples of Controversial & Implausible Premises**

The amount of evil in our world has no effect on the probability of whether or not God exists.

Female students at Auckland University have better grades (on average) than male students.

It is obvious that the Prime Minister was informed about the early release of an OIA request to Cameron Slater.

# Not all implausible premises are controversial

Everything is made out of water.

There are 73 pink elephants in this room.

Rubber cakes taste better than chocolate cakes.

# Some Laws of Logic

Let 'P' and 'C' be placeholders for arbitrary propositions.

## First law

$P \ \& \ C$  entails  $C$

## Example

I have had a hot chocolate and I have had a flat white.

entails that:

I have had a flat white.

# Second Law

If  $P$  entails  $C$ , then  $C$  is at least as probable as  $P$ .

## Example

I had a banana this morning

entails that:

I had some fruit this morning.

So, the probability that I had some fruit this morning is at least as great as the probability that I had a banana this morning.

# Why?

Having a banana is enough for having some fruit.

So the chance that it's true that I had some fruit can't be less than the chance that I had a banana.

Is having some fruit as probable as having a banana?

No, because there are other ways of having some fruit that don't involve having a banana.

# Third Law

If  $P$  entails  $C$ , then  $P$  and not- $C$  are inconsistent.

## Example

There are four people in this room.

entails that:

There are people in this room.

Which, in turn, means that we cannot also say:

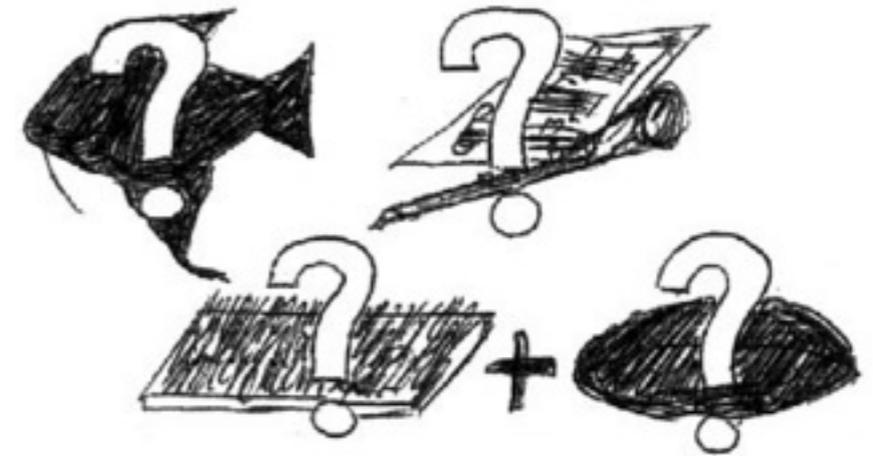
There are no people in the room.

a. Bruce is a marine biologist

b. Bruce is a carpet layer

c. Bruce plays the oboe

d. Bruce is a rugby coach and a carpet layer



Most people say:

‘Bruce is a rugby coach and Bruce is a carpet layer’ is more probable than ‘Bruce is a carpet layer.’

But most people are wrong about that. Here's why.

The first law of logic said:

$P \ \& \ C \text{ entails } C$

So by this first law:

Bruce is a rugby coach and Bruce is a carpet layer entails that:

Bruce is a carpet layer

That is,

(d) entails (b)

Meanwhile, the second law said:

If P entails C, then C is at least as probable as P.

By this second law:

Bruce is a carpet layer

is at least as probable as:

Bruce is a rugby coach and a carpet layer

That is,

(b) is at least as probable as (d).

But then, it cannot be that (d) is more probable than (b).

Hence, most people are wrong to say that (d) is more probable than (b).